Finding spin glass ground states using multi-stage quantum walks

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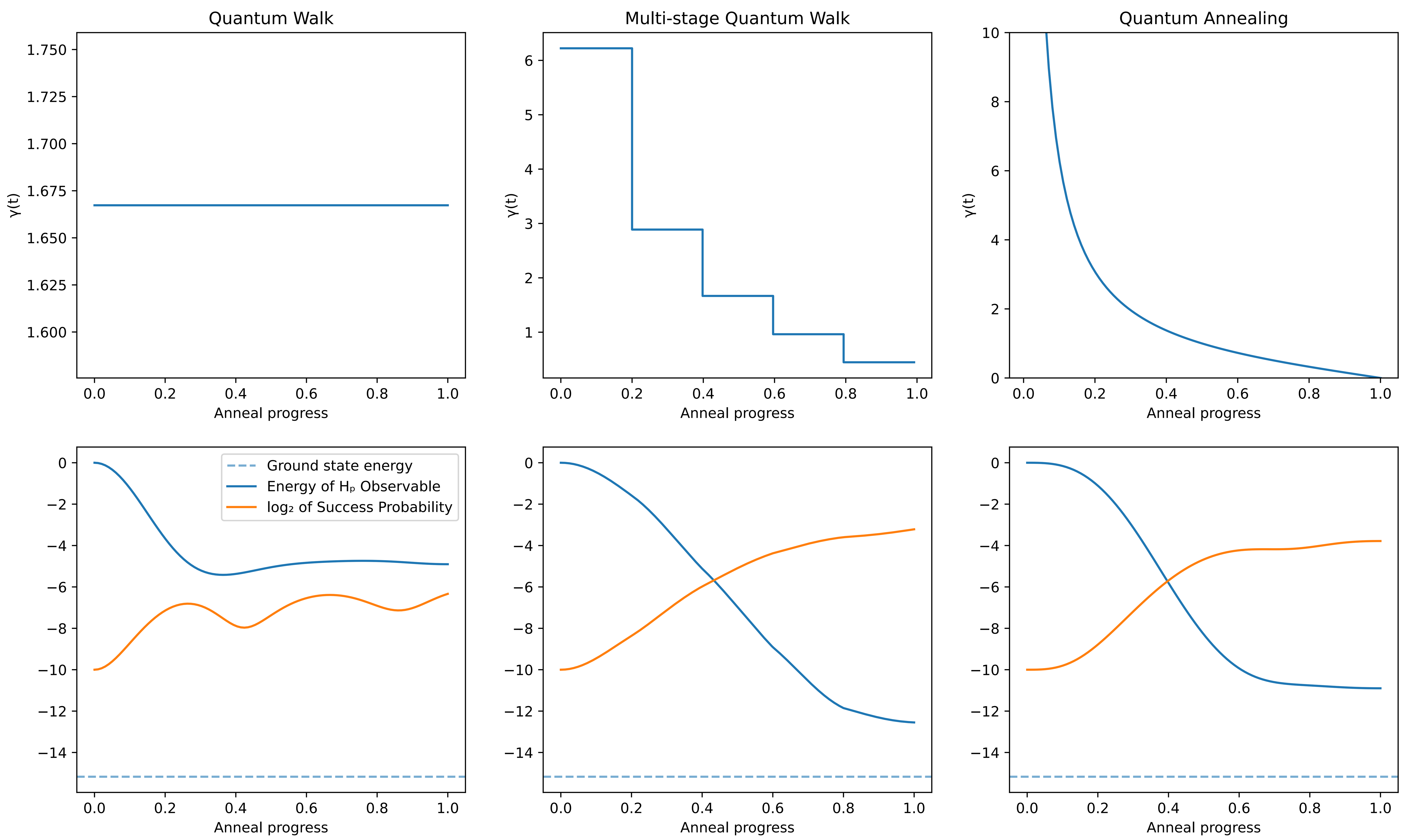
https://github.com/Asa-Hopkins/Multistage-QW

Solving problems with quantum annealing:

1. Encode the problem's solution as the ground state of a Hamiltonian $\mathbf{H}_P$
2. Prepare a register of qubits in the ground state of another Hamiltonian $\mathbf{H}_G$, which is chosen so that this is easy
3. Evolve the system under the Hamiltonian $A(t)\mathbf{H}_P + B(t)\mathbf{H}_G$
4. Measure the system, and hope it gives the optimal solution

To make things simpler, we work with the ratio $\gamma(t) = \frac{B(t)}{A(t)}$, leaving the energy scale to be defined by the hardware. Multi-stage quantum walks can then be implemented as a quantum anneal where $\gamma(t)$ is piecewise constant.

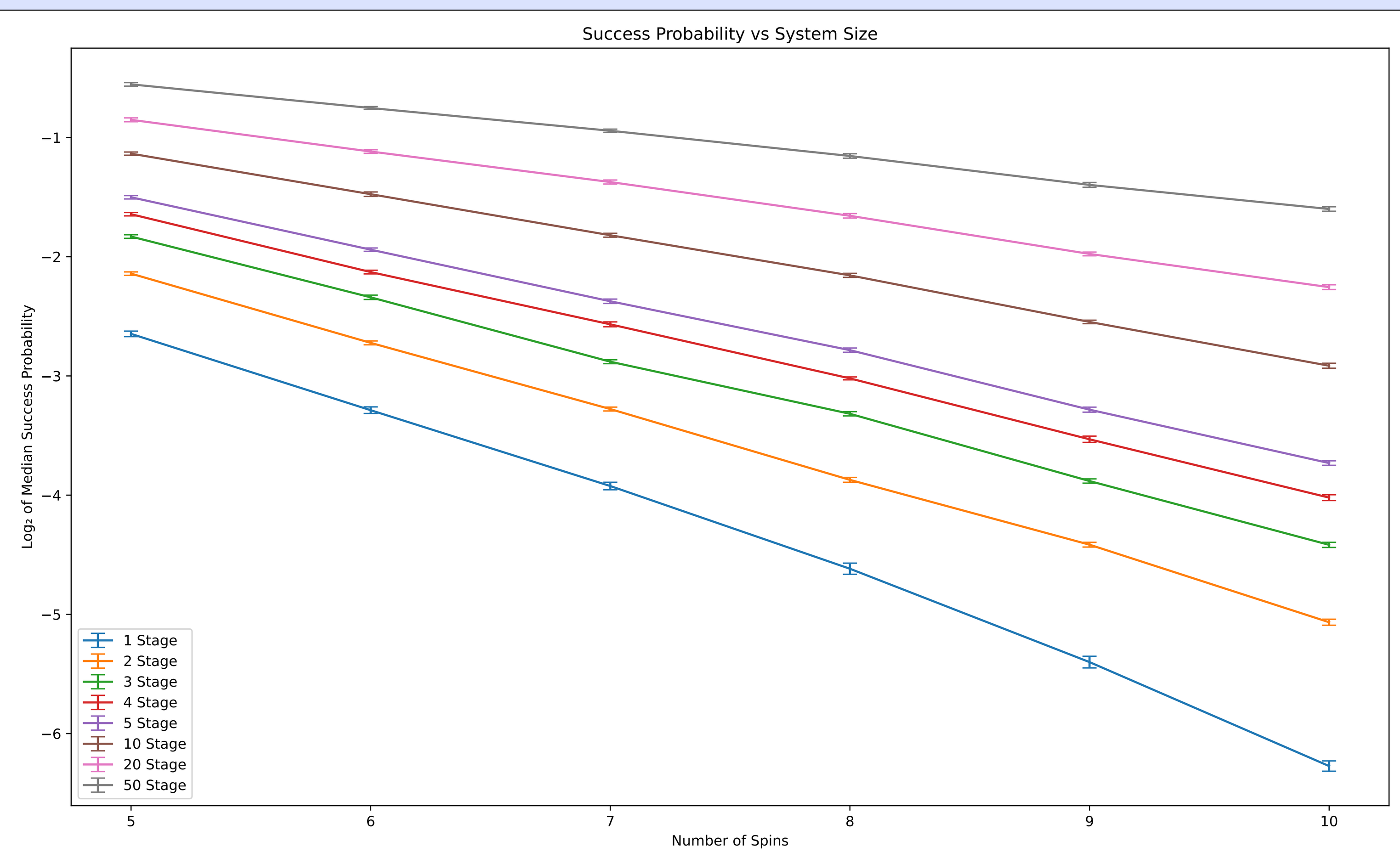


When to measure?

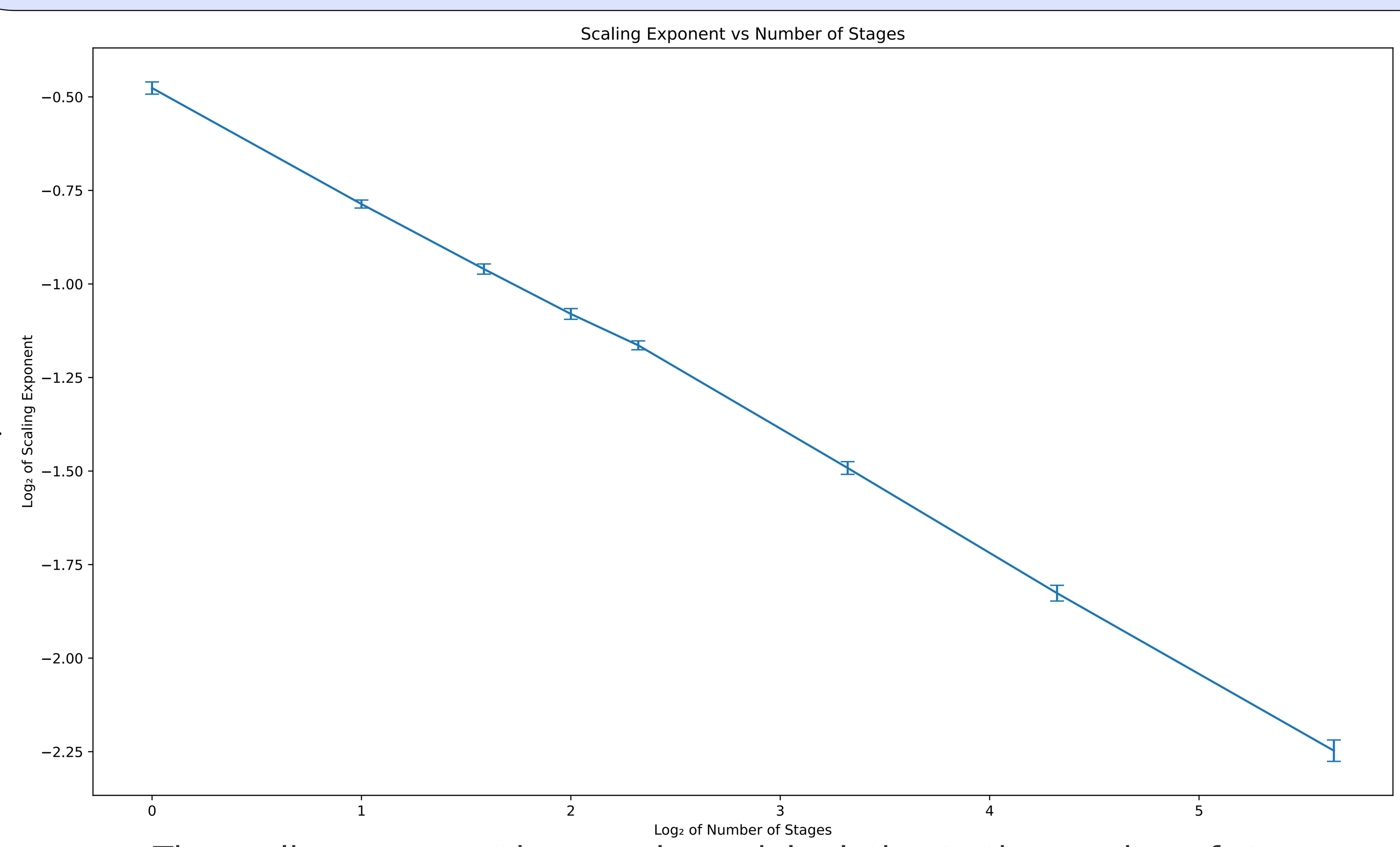
To average out the oscillating succes rate, it is necessary to sample at multiple times, but what times are best? There is a 'warm up' time where the state energy is decreasing rapidly, and a Taylor expansion shows that this decrease is initially quadratic. Since the energy always starts at 0 and can never go below the true ground state which is $O(n)$ [1], then after a time $O(\sqrt{n})$ the rapid decrease must have finished.

How to choose $\gamma(t)$?

In quantum annealing, it is known that for any given problem there is a choice of $\gamma(t)$ that solves the problem in linear time. Finding this perfect γ for each problem is sadly infeasible so heuristics are used to choose reasonable but suboptimal values. The 'infinite time' success chance suggests that we want each stage to rotate the state a fixed amount towards the end state. To achieve this we therefore 'rotate' the Hamiltonian and choose $\cos(\frac{n\pi}{2(m+1)})\mathbf{H}_G + \sin(\frac{n\pi}{2(m+1)})\mathbf{H}_P$ for the n^{th} stage out of m.



The problem becomes exponentially more difficult as more spins are added, but the difficulty grows more slowly for more stages.



The scaling exponent has a polynomial relation to the number of stages, leading to a polynomial algorithm for solving the spin glass problem [2].

PhD supervisor: Viv Kendon

[1] Adam Callison et al. *Finding spin glass ground states using quantum walks.* https://arxiv.org/abs/1903.05003
[2] Aditi Misra-Spieldenner et al. *Mean-Field Approximate Optimization Algorithm.* https://arxiv.org/abs/2303.00329

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